Closing Today: 2.2
Closing Monday: 2.3
Closing Wed: 2.5-6
Closing next Fri: 2.7

My extra office hours today
1:15-3:00pm in
Communications B-006
(room next to MSC)

Entry Task: Find the limits

1. $\lim_{h \to 0} \left[\frac{(5+h)^2 - 5}{h} \right]$

2. $\lim_{x \to 2^-} \left[\frac{5x+1}{x^2 - 4} \right]$

3. $\lim_{x \to 16} \left[\frac{x - 16}{\sqrt{x} - 4} \right]$

2.5 Continuity

A function, $f(x)$, is **continuous at x = a** if

$$\lim_{x \to a} f(x) = f(a)$$

this implies three things

1. f(a) is defined,
2. $\lim_{x \to a} f(x)$ exists and is finite, and
3. they are the same!

Continuous from the left

$$\lim_{x \to a^-} f(x) = f(a)$$

Continuous from the right

$$\lim_{x \to a^+} f(x) = f(a)$$

Casually, we might say a function is continuous at *x = a* if you can draw the graph across *x = a* point without picking up your pencil.

The "standard" precalculus functions are **continuous everywhere they are defined**:

polynomials → defined everywhere
sin(x), cos(x) → defined everywhere
e^x → defined everywhere
odd roots → defined everywhere
$\tan^{-1}(x)$ → defined everywhere
Rational Functions → for denom $\neq 0$
Even Roots → under radical ≥ 0
ln(x) → for $x > 0$
tan(x) → not at $x = \pm k\pi/2$
$\sin^{-1}(x)$, $\cos^{-1}(x)$ → for $-1 \leq x \leq 1$

All the functions above are continuous everywhere accept at the noted exceptions.

Example:

$$g(x) = \begin{cases} 8 - x^2 & , \text{if } x < 0; \\ 2 & , \text{if } 0 \leq x < 5; \\ 0 & , \text{if } x = 5; \\ 7 - x & , \text{if } x > 5. \end{cases}$$

Does g(x) have any discontinuities?

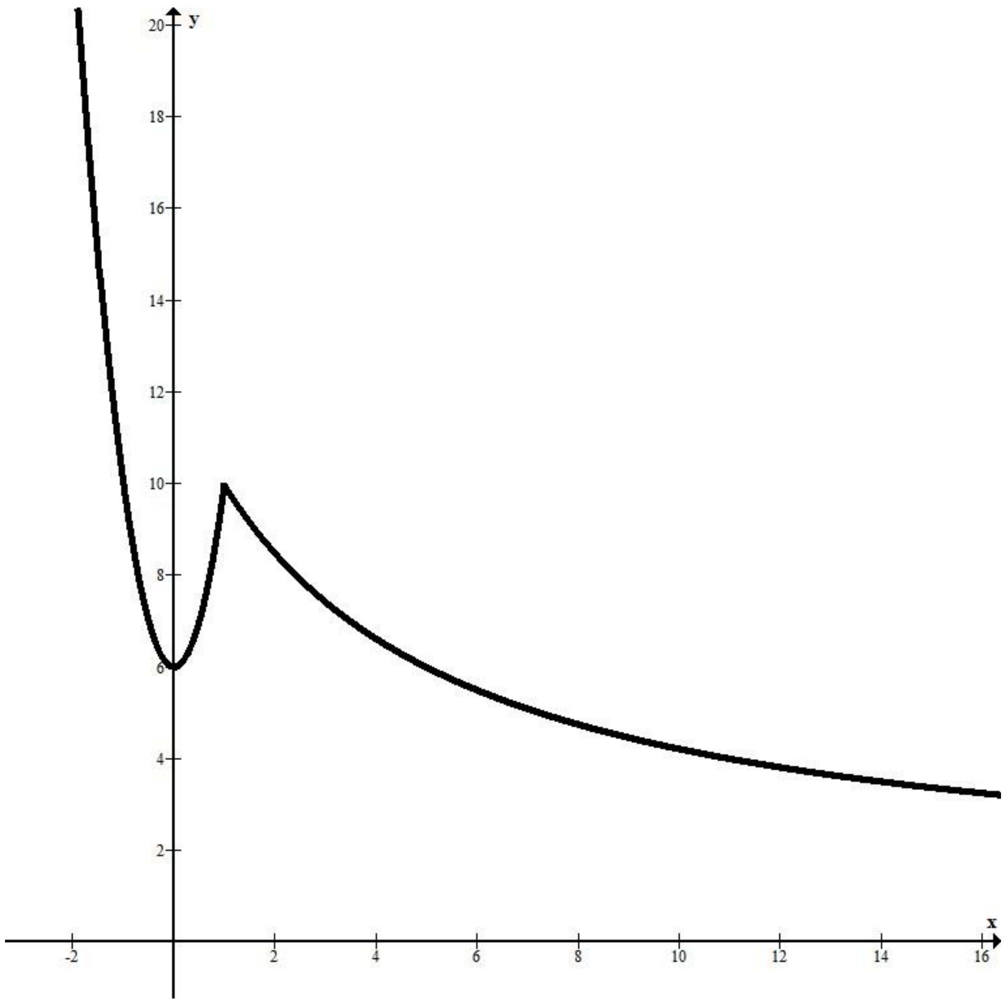
If so, where?

Example:

$$h(x) = \begin{cases} ax^2 + 6 & \text{, if } x < 1; \\ b & \text{, if } x = 1; \\ \dfrac{x + 49}{x + a} & \text{, if } x > 1. \end{cases}$$

Find the values of *a* and *b* that will make $h(x)$continuous *everywhere*.

h(x)



Theorem:

If $f(x)$ is continuous at $x = b$, and

$$\lim_{x \to a} g(x) = b$$

then

$$\lim_{x \to a} f\big(g(x)\big) = f(b).$$

Example:

Find

$$\lim_{x \to 9} \ln\left(\frac{\sqrt{x} - 3}{x - 9}\right)$$

2.6 Limits "at" Infinity
(*Horizontal Asymptotes*)

Goal: Study "long term" behavior.

When we write

$$\lim_{x \to \infty} f(x) = L$$

we say "the limit of f(x), as x goes to infinity is L",
and we mean

as *x* takes on larger and larger positive numbers,
y = f(x) takes on values closer and closer to L.

Similarly,

$$\lim_{x \to -\infty} f(x) = L$$

we say "the limit of f(x), as x goes to negative infinity is L".

Some important limits to know:

For any <u>positive</u> number *n*,

1. $$\lim_{x\to\infty} x^{-n} = \lim_{x\to\infty} \frac{1}{x^n} = 0.$$

$$\lim_{x\to-\infty} x^{-n} = \lim_{x\to-\infty} \frac{1}{x^n} = 0.$$

(if defined)

2. $$\lim_{x\to\infty} e^x = \infty \quad \text{and} \quad \lim_{x\to\infty} e^{-x} = 0.$$

$$\lim_{x\to-\infty} e^x = 0 \quad \text{and} \quad \lim_{x\to-\infty} e^{-x} = \infty$$

3. $$\lim_{x\to\infty} \ln(x) = \infty$$

4. $$\lim_{x\to\infty} \tan^{-1}(x) = \frac{\pi}{2},$$

$$\lim_{x\to-\infty} \tan^{-1}(x) = -\frac{\pi}{2}$$

Strategies to compute
$$\lim_{x \to \infty} f(x)$$

1. Is it a known limit?

2. **Rewrite it in terms of known limits**:

Strategy 1: Multiply top/bottom by $\frac{1}{x^a}$, where *a* is the largest power.

Strategy 2: Multiply top/bottom by e^{-rx}.

Strategy 3: Multiply by conjugate.

Strategy 4: Combine Fractions.

Strategy 5: Change variable